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Calculation of Laminar Boundary Layers on Continuous Surfaces by Meksyn's Method

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In the case of laminar boundary layers on continuous surfaces, Eickhoff[1,2] has shown that the linearization of the boundary-layer equations leads to analytical results for the heat-transfer rates that agree well with the numerical results of Rhodes and Kaminer[3] over a wide range of Prandtl numbers ($0.1 \leq \sigma \leq 1000$). In general, this method can also be applied for the analysis of the shock tube boundary layers. In fact, this was done some years ago by the authors[4,5] and the results obtained were compared with the numerical results of Mirels.[6] The purpose of this Note is to present some important results obtained by the authors. It may be noted that these results agree with those of Eickhoff when the shock strength $\lambda \gg 1$; they can also be used for values of λ relevant to shock tube boundary layers ($1 \leq \lambda \leq 6$ for $\gamma = 1.4$).

Following Mirels and the usual notation, the governing equations and the boundary conditions for solving the laminar boundary layer which develops on an uninsulated wall behind a shock wave advancing into a stationary fluid are

$$f''' + ff'' = 0 \tag{1}$$

$$s'' + \sigma f s' = 0 \tag{2}$$

$$f(o) = 0, \quad f'(o) = \lambda, \quad f'(\infty) = 1 \tag{3}$$

$$s(o) = 1, \quad s(\infty) = 0 \tag{4}$$

Using Meksyn's technique,[7] Eqs. (1–4) can be solved[4] for the two unknowns $f''(o)$ and $s'(o)$—the two important wall derivatives the former relating to wall shear and the latter to wall heat transfer. By integrating Eqs. (1) and (2) from zero to infinity and then substituting the boundary conditions (3) and (4), we get

$$1 - \lambda = \alpha \int_0^\infty e^{-F}\, d\eta \tag{5}$$

and

$$-1 = k \int_0^\infty e^{-\sigma F}\, d\eta \tag{6}$$

where

$$\alpha = f''(o), \quad k = s'(o) \quad \text{and} \quad F = \int_0^\eta f\, d\eta$$

For $\eta \to 0$, the series expansion for F is

$$F = \frac{\lambda \eta^2}{2!} + \frac{\alpha \eta^3}{3!} - \frac{\alpha \lambda \eta^5}{5!} + \cdots \tag{7}$$

and by inversion,

$$\eta = A_0 F^{1/2} + (A_1/2)F + (A_2/3)F^{3/2} + \cdots \tag{8}$$

where $A_0 = (2/\lambda)^{1/2}$, $A_1 = -(2\alpha/3\lambda^2)$, $A_2 = (5/6)(\alpha/\lambda^2)^2(\lambda/2)^{1/2}$, etc. After integrating Eqs. (5) and (6) using Eq. (8), we get

$$1 - \lambda = \alpha(B_0 + B_1\alpha + B_2\alpha^2 + \cdots) \tag{9}$$

and

$$-1 = k\left(\frac{B_0}{\sigma^{1/2}} + \frac{B_1}{\sigma}\alpha + \frac{B_2}{\sigma^{3/2}}\alpha^2 + \cdots\right) \tag{10}$$

where $B_0 = (\pi/2\lambda)^{1/2}$, $B_1 = -(1/3\lambda^2)$, $B_2 = (5/24)(1/\lambda^4)(\pi\lambda/2)^{1/2}$; etc., α and k may be evaluated from Eqs. (9) and (10), respectively. Hsu[8] also has obtained the value of α using Meksyn's technique and he gives a more general form of Eq. (9) by retaining higher order terms in the series expansion (7). However, even the first three terms on the RHS of Eq. (9) provide a good approximation and α can be calculated by solving the cubic equation

$$\alpha^3 + p\alpha^2 + q\alpha + r = 0 \tag{11}$$

where

$$p = -\frac{8}{5}\left(\frac{2}{\pi}\right)^{1/2}\lambda^{3/2}, \quad q = \frac{24}{5}\lambda^3, \quad r = \frac{24}{5}\left(\frac{2}{\pi}\right)^{1/2}\lambda^{7/2}(\lambda - 1)$$

It may be easily verified that there is only one real root of α which is of interest to the present problem and the other two are complex. If only the first two terms in the series (8) are retained, the resulting equation for α is a quadratic and the solution is obtained in a closed form as

$$\alpha = \left(\frac{9\pi}{8}\right)^{1/2}\lambda^{3/2}\left[1 - \left\{\frac{8+3\pi}{3\pi} - \frac{8}{3\pi\lambda}\right\}^{1/2}\right] \tag{12}$$

Once α is known, k can be easily determined from Eq. (10). Taking only the first three terms on the RHS of Eq. (10), we get

$$k = \frac{-\sigma^{1/2}}{\left(\frac{\pi}{2\lambda}\right)^{1/2} - \frac{\alpha}{3\lambda^2\sigma^{1/2}} + \left(\frac{5}{24}\right)\left(\frac{\alpha}{\lambda^2}\right)^2\left(\frac{1}{\sigma}\right)\left(\frac{\pi\lambda}{2}\right)^{1/2}} \tag{13}$$

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The values of $f''(o) = \alpha$ and $s'(o) = k$ as calculated from Eqs. (11–13) for different values of shock strength λ are compared in Table 1 with the numerical results of Mirels.

Table 1 Values of $f''(o)$ and $s'(o)$ for different λ

	$f''(o)$			$s'(o)$ for $\sigma = 0.72$	
λ	Eq. (12)	Eq. (11)	Mirels[6]	Eq. (13)	Mirels[6]
2	−1.0288	−1.0065	−1.0191	−0.8339	−0.8512
4	−4.2007	−4.0330	−4.0623	−1.0995	−1.1156
6	−8.4729	−8.0812	−8.1009	−1.3165	−1.3262

For vanishing velocity at the outer edge of the boundary layer or for large shock strengths (i.e., $\lambda \gg 1$), the equations for $f''(o)$ and $s'(o)$ given by Eickhoff are the same as Eqs. (11) and (13). For large values of σ, it can be shown that

$$s'(o) \approx -(2\lambda\sigma/\pi)^{1/2}$$

The Nusselt number (for $\sigma \to \infty$) is given by

$$Nu = -\frac{1}{(2\lambda)^{1/2}}(Re)^{1/2}s'(o) = \frac{1}{(\pi^{1/2})}(\sigma\, Re)^{1/2} \tag{14}$$

The preceding method can also be applied to the case when the wall is insulated. Following Mirels, the recovery factor $r(o)$ is expressed in quadrature form as

$$r(o) = \frac{2\sigma\alpha^2}{(\lambda-1)^2}\int_\eta^\infty e^{-\sigma F}\,d\xi \int_0^\xi e^{-(2-\sigma)F}\,d\theta \tag{15}$$

For $\sigma < 2$, the inner integral in Eq. (15) can be expressed as a sum of incomplete gamma functions when the series expansion (8) is substituted for the independent variable. If we retain only the first term in the series expansion (8) and, to be consistent with this approximation, use only the first term on the RHS of Eq. (9) to evaluate α, then Eq. (15) integrates to

$$r(o) = \frac{4}{\pi}\left(\frac{\sigma}{2-\sigma}\right)^{1/2} \sin^{-1}\left(\frac{2-\sigma}{2}\right)^{1/2} \tag{16}$$

Thus, to the first order, the recovery factor does not depend on the shock strength at all; instead it depends only on the Prandtl number. For $\sigma = 0.72$, $r(o) = 0.8855$. It may be noted here that Eq. (16) is the same as that given by Emmons[9] for the recovery factor of an accelerated plate starting from rest. The result for $r(o)$ can be improved by higher approximations and this has been done in Refs. 4 and 5 by retaining the first two terms in the series expansion for F. However, $r(o)$ was found to have a weak dependence on the shock strength. Numerical results of Mirels also show a similar weak dependence.

As mentioned by Eickhoff, these analytical solutions show general tendencies. Also, the simplicity of the solution obtained by one-term approximation suggests the following empirical relations for α and k:

$$\alpha = -(0.211 + 0.575\lambda^{1/2})(\lambda - 1) \tag{17}$$

and

$$k = -(0.202 + 0.459\lambda^{1/2}) \quad \text{for} \quad \sigma = 0.72 \tag{18}$$

The results presented here have the advantage of being applicable over a wide range of values of λ and σ.

After determining the wall derivatives $f''(o)$ and $s'(o)$ which are of major interest, it is possible to calculate the velocity and temperature profiles.[4] The calculations indicate that the free-stream conditions are approached earlier than in the exact solutions.

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Technical Comments

Comment on "Criteria for Selecting Curves for Fitting to Data"

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I. Introduction

IN an Aug. 1970 paper,[1] T. J. Dylewski proposed an interesting criterion for discriminating between "oversmoothing" and "undersmoothing" in the least squares fitting of equations to data. This criterion is given by

$$B = S'/S''$$

where S' is the sum of squared deviations from fitting a given curve to all the data and S'' is the pooled sum of squared deviations from fitting a curve of the same type to each of two halves of the data (for simplicity, let the sample size, N, be an even integer). Among the advantages the author cites for this criterion is that it does not permit tests of significance. Before showing that it actually does, and indeed that an evaluation of significance is a necessity, it may be well to comment on statistical tests.

II. Statistical Tests

The author uses the term, test of significance, to describe the process of choosing a Type I Error risk (level of significance), calculating a test statistic and then either accepting or rejecting the hypothesis being considered. There has been some effort, though not entirely successful, in the statistical literature to refer to this procedure as hypothesis testing, or more explicitly, accept/reject decision-making. While hypothesis testing seems to have direct applicability to acceptance sampling, where indeed one is required to make a series of accept/reject decisions, and perhaps to certain confirmatory experiments, it is of questionable value in

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